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The instability parameters of stellar atmospheres

ABSTRACT

We discuss the significance of the adiabatic exponents Γ_1, Γ_2 and Γ_3 and derive expressions for their calculation in a stellar atmosphere including simultaneous single-ionization of various elements and the presence of an equilibrium radiation field. A discussion is given of the relation of Γ_1 to dynamic instability. It is shown that some parts of some Kurucz models for extreme supergiant atmospheres are dynamically instable as a result of ionization and radiation in the deeper layers.

INTRODUCTION

We consider a mixture of various elements in thermal and chemical equilibrium, without hydrodynamic motions. The interactions between the radiation and the gas are neglected.

The generalized adiabatic exponents $\Gamma_{1,2,3}$ are useful for the study of adiabatic processes in an ionizing gas. They are related to the behaviour of thermodynamic systems under infinitesimal isentropic changes. Γ_1 is important in connection with dynamical instability in stars and is defined by

$$\Gamma_1 \equiv \left(\frac{\partial \ln P}{\partial \ln \rho}\right)_S , \qquad (1)$$

where P is the sum of gas- and radiation pressure, and ρ is the mass-density. Γ_2 is important for the study of convective instability, and Γ_3 for pulsational instability of stars.

BEHAVIOUR OF Γ_1 IN IONIZATION REGIONS

Γ_1 varies between $\frac{4}{3}$ and $\frac{5}{3}$, for a mixture of non-ionizing monoatomic gas and black-body radiation, depending on the fraction β of the gaspressure to the total pressure. In this case Γ_1 can not decrease below $\frac{4}{3}$. However, when ionization is also included in the system, this becomes possible. When single ionization of various elements is considered, we derived expressions for Γ_1, using the specific heats for this system, derived by Mihalas (1965). We find,

$$\Gamma_1 = \frac{C_p}{C_v} \frac{\beta(\bar{x}^2 + \bar{x} + \sum_i \nu_i x_i(1 - x_i))}{(1 + \bar{x})(\bar{x} + \sum_i \nu_i x_i(1 - x_i))} , \qquad (2)$$

where ν_i denotes the element abundance, x_i the degree of ionization of each element, and $\bar{x} = \sum_i \nu_i x_i$ the mean degree of ionization. Eq.(2) reduces to the result of Cox (1968) for a gas consisting of one ionizing element with radiation.

In fig.1 we plot Γ_1 against absolute temperature for three different electronpressures, using solar abundance of 16 elements in the system. At low temperatures the gaspressure is higher than the radiationpressure, hence $\Gamma_1 \cong \frac{5}{3}$. For high temperatures, radiationpressure overtakes $P_{\mathbf{gas}}$, and hence $\Gamma_1 \cong \frac{4}{3}$.

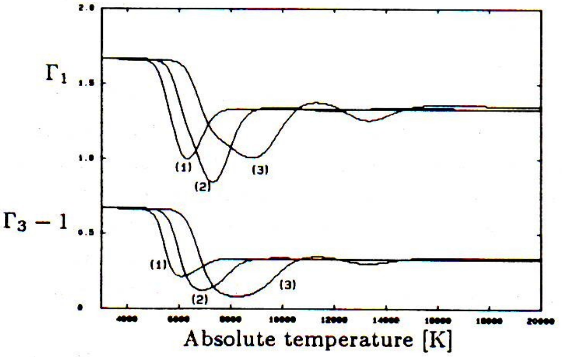

Figure 1: Γ_1 and $\Gamma_3 - 1$ for a system with ionization and radiation with electronpressure (1) P_e=0.1, (2) P_e=1.0, (3) P_e=20.0

The first depression in the curve of Γ_1 is due to the ionization of H (HI→HII), while the second is the result of single ionization of He (HeI→HeII). In these regions most of the compression energy is transformed into ionization energy, rather than into kinetic energy of thermal motion, so that the temperature will increase less upon compression than it would in a neutral or fully ionized zone. Since the increase of the kinetic energy of the particles (ions *and* released electrons) is low as compared to the increase in mass-density upon compression, Γ_1 can have values below $\frac{4}{3}$. This indicates the high compressibility of the gas in the ionization regions.

We find that Γ_1 can even have values below unity (fig.1) in certain ranges of electronpressure and temperature, resulting from the combination of ionization *and* radiation. This effect appears for 0.1 $\frac{\text{dyn}}{\text{cm}^2} \le$

$P_e \leq 20.0\ \frac{\text{dyn}}{\text{cm}^2}$ and $6100\ \text{K} \leq T \leq 9000\ \text{K}$ in the hydrogen ionization region for solar abundance. Theoretically, it can be shown that $\Gamma_1 < 1$ does not correspond to the situation of a negative polytrope. We also notice that in many theoretical deductions for shock waves, Γ_1 is expected to have only values above unity (cfr. $S^2_{\text{ad}} = \frac{P}{\rho}\Gamma_1$ should be used and not $\frac{P}{\rho}\frac{C_p}{C_v}$, where $\frac{C_p}{C_v} > 1$). Physically, this corresponds to the production of strong dynamical instabilities under perturbations in stellar atmospheres.

APPLICATION TO THE KURUCZ MODELS

Ledoux (1965) showed that a star becomes instable when Γ_1 falls below $\frac{4}{3}$ throughout an appreciable fraction. He also found that regions where Γ_1 is lower than $\frac{4}{3}$ and decreases with increasing radius, have a destabilizing influence. The potential energy of contracting layers will mainly be transformed into ionization energy, without sufficiently increasing the pressure in the layers to stop the movement and restore the equilibrium.

With eq.(2) we calculated Γ_1 throughout the atmospheres of several Kurucz models of extreme supergiants with T_{eff} between 5500 K and 20000 K, each with the lowest gravitational acceleration presented in the tables. This is shown in fig.2 for T_{eff} between 5500 K and 8000 K, where we take only gaspressure into account. Γ_1 is gradually decreasing in the outer parts of the atmosphere because the H ionization zone is moving outwards for higher temperatures. Notice also the depression in Γ_1 in the deeper layers, which is due to single ionization of He.

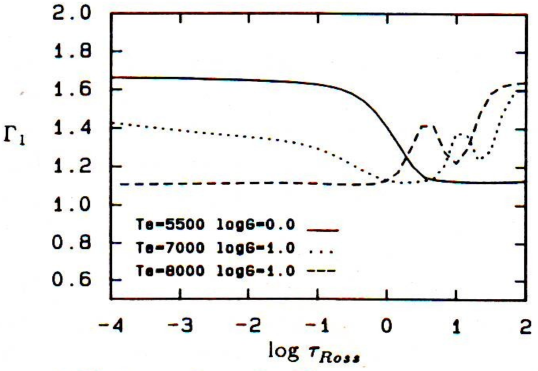

Figure 2: Γ_1 throughout the Kurucz model atmospheres of extreme supergiants with T_{eff}= 5500, 7000 and 8000 K.

When T_{eff} has increased to 13000 K, in fig.3, Γ_1 increases again, which indicates that all hydrogen is already ionized at the surface.

From fig.4 we notice that the ionization of He also reduces Γ_1 to below $\frac{4}{3}$ for models with T_{eff} between 15000 K and 20000 K. Single ionization of He has a similar local destabilizing effect as the ionization of H over a large part of the atmosphere of extreme supergiants. T_{eff} has to be increased up to 20000 K to find Γ_1 -values above $\frac{4}{3}$ over the whole atmosphere. In these atmospheres He is completely singly ionized.

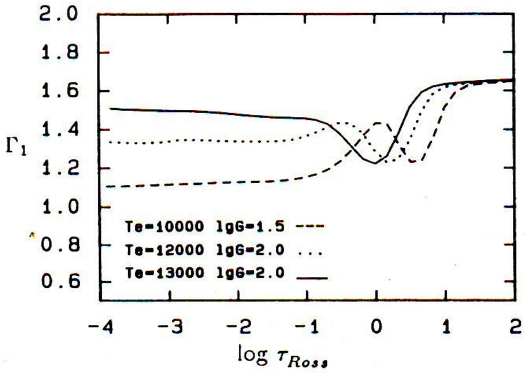

Figure 3: Γ_1 throughout the Kurucz model atmospheres with T_{eff}= 10000 K, 12000 K, 13000 K.

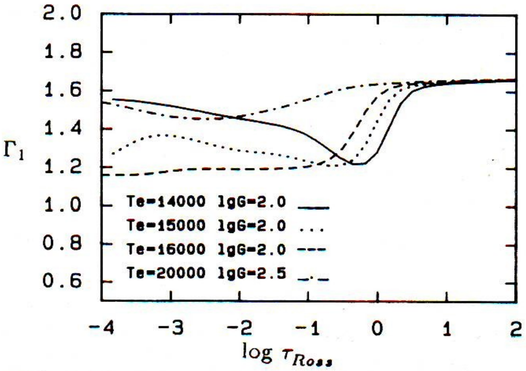

Figure 4: Γ_1 throughout the Kurucz model atmospheres with T_{eff} between 14000 K and 20000 K.

When black-body radiation is included in the calculations, we find that Γ_1 obtains values that are only slightly larger than $\frac{4}{3}$ in the deeper layers (instead of $\approx \frac{5}{3}$ for $\log \tau_{Ross} > 1$ in fig.3 and 4, where radiation is neglected), which contributes to the instability of the atmosphere. In addition, we find that for the model atmosphere with $T_{\text{eff}} = 8000$ K and $\log g = 1.0$ the value of Γ_1 can drop very steeply to below unity at fairly high optical depths ($\tau_{\text{Ross}} \cong 1$), which enhances the origin of shock waves.

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